MINISTERUL EDUCAȚIEI NAȚIONALE

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MATH-Individual Contest
SUBJECTS-CLASA a IX a

Problem 1

If $a,b,c \in R_+^*$ si $a+b+c=1$ prove:

a). $(a+bc)(b+ac)(c+ab)=(a+b)^2(b+c)^2(c+a)^2$

b). $\left(\frac{a}{bc}+1\right)\left(\frac{b}{ac}+1\right)\left(\frac{c}{ab}+1\right) \geq 64$

Problem 2

Be M a point inside ΔABC non-isosceles of sides a,b,c with G the barycenter (center of gravity) of the triangle and I the center of the incircle.

a)Demonstrate that $\frac{AM}{MD}=\frac{A_{\Delta ABM}+A_{\Delta ACM}}{A_{\Delta BCM}}$ where $\{D\}=AM \cap BC$.

b)Demonstrate that M,G,I are collinear if and only if

$(b-c)A_{\Delta BCM}+(c-a)A_{\Delta CAM}+(a-b)A_{\Delta ABM}=0$.

Problem 3

Be $n \in N^*$ determined.

a)Determine the rest of the division of 5^{2^n} to 2^{n+2}.

b)Show that the array (order) $a_n=\left(\left\{\frac{5^{2^n}}{2^{n+2}}\right\}\right)^{-1}$, where $\{a\}$ is the fractionary part of the real number a, is a geometric progression and that every other arithmetic progression $(b_n)_{n\geq 1}$ with natural terms that has two common terms with $(a_n)_{n\geq 1}$ has an infinity of common terms with the progression $(a_n)_{n\geq 1}$.

Problem 4

Consider the array $(x_n)_{n\geq 0}$ given by $x_n = 2^n - n$, $n \in N$. Determine all the natural numbers p for which $s_p = x_0 + x_1 + x_2 + \ldots + x_p$ is a degree with a natural exponent of 2.

Note:

The available time for solving the subjects is 3 hours

Each subject is worth 25 points.