## CONCURSUL PLURIDISCIPLINAR PROSOFT@NT - JUNIOR

martie 2017

## MATHEMATICS-INDIVIDUAL
## $8^{TH}$ GRADE SOLUTIONS

**SUBJECT 1 :**

a) Let $x, y \in \mathbb{R}$, $x-5y+3=0$ şi $x \in [-3;2]$. Calculate

$$a=\sqrt{x^2+y^2+6x+9}+\sqrt{x^2+y^2-4x-2y+5}.$$

b) Let a, b, c positive, rational numbers so that $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$. Demonstrate that $\sqrt{a^2+b^2+c^2} \in \mathbb{Q}$.

**SUBJECT 2 :**

Let the expression $E(x)=\left(\frac{7x-3}{2x^2-3x+1}+\frac{2}{1-x}\right):\frac{6x^2+x-1}{x-1}$, $x \in D$, where $D$ is the maximum range of definition of the expression.

a. Demonstrate that $E(x)=\frac{1}{4x^2-1}$, $x \in D$.

b. Calculate $S=E(2)+E(3)+...+E(2017)$

**SUBJECT 3:**

Let $SABCD$ a regular quadrilateral pyramid.

$AM \perp SB, M \in SB, BN \perp SC, N \in SC, CP \perp SD, P \in SD, DQ \perp SA, Q \in SA$ and $R$ the symmetric of $N$ towards $AC$.

a) Demonstrate that $B, R, Q, D$ are coplanar.

b) Find the size of the angle between the straight lines $MP$ and $RQ$.

**SUBJECT 4 :**

Let $G_1$ and $G_2$ the barycentres of the traingles ACD, BCD respectively , the traingles situated in different plans. We consider N the midpoint of the segment [CD], M∈(AB) so that $\frac{AM}{AB}=\frac{2}{5}$ şi $MN \cap AG_2=\{E\}$.

Demonstrate $EG_1 \parallel (BCD)$.

**Note:**

The available time for solving the subjects is 3 hours

Each subject is worth 25 points.