MULTIDISCIPLINARY PROSOFT@NT CONTEST

March 2018

PROBLEMS

XI TH GRADE - MATHEMATICS

PROBLEM I

Let $k \geq 2$ a given natural even number and the function $f:\mathbb{R}\to\mathbb{R}$, $f(x)=\dfrac{C_k^1x+C_k^3x^3+...C_k^{k-1}x^{k-1}}{C_k^0+C_k^2x^2+...+C_k^kx^k}$.

Calculate $\lim\limits_{n\to\infty}\left(\underbrace{f\circ f\circ...\circ f}_{n\text{ ori}}\right)(x)$, where $x\in\mathbb{R}$. ***

PROBLEM II

Let matrices $A,B,C\in\mathbf{M}_n(\mathbb{R})$, such that $ABC=I_n$. Prove that, if matrices

$I_n+A+AB, I_n+B+BC, I_n+C+CA$ are invertible, then the sum of their inverses equals I_n.

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PROBLEM III

Calculate: $\lim\limits_{n\to\infty}\dfrac{1}{(n!)^{\frac{1}{n}}}\cdot\log_2\left(\sqrt[2^n]{2}+\sqrt[2^n]{3}+.....+\sqrt[2^n]{2^n+1}\right)$ ***

PROBLEM IV

Determine the functions $f:[0,3]\to(0,1]$ which respect/follow the conditions:

i. $\lim\limits_{x\to0}\dfrac{f(x)-f(0)}{x^2}\in\mathbb{R}$

ii. $f(3x)+3f(x)=4f^3(x),\ \forall x\in[0,1]$ ***

Note

Time: 3 hours.

Each problem solved correctly is worth 25 points.